Semiconductor laser Markov models in the micro-canonical, canonical and grand-canonical ensembles

Arthur Vallet, Laurent Chusseau, Fabrice Philippe, Alain Jean-Marie

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A semiconductor laser Markov picture

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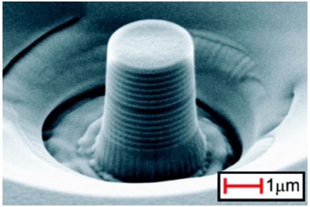
11/07/17

Summary

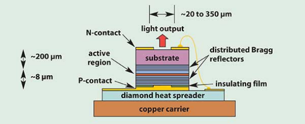
Summary

Semiconductor Laser overview



Micropillar with embedded colloidal CdSe/ZnS quantum dots

- $\approx 1\ \mu m^2$
- $\approx 10\ \mu W$



Vertical-Cavity Surface-Emitting Laser

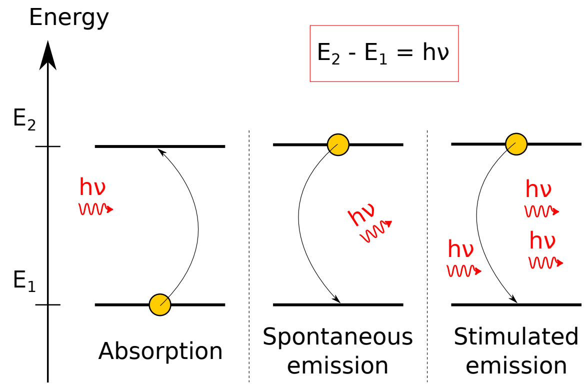
- $\approx 0.01\ mm^2$
- $\approx 1\ mW$

High power VECSEL

- $\approx 1\ cm^2$
- $\approx 1.5\ kW$

[1] R. Bratschitsch and al., "Advances in Lasers and Electro Optic." *InTech*, 2010.

Photonics transitions

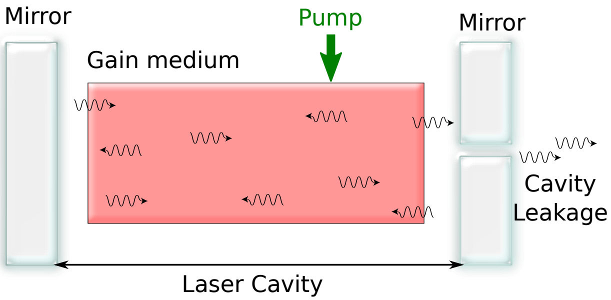


Photonics transitions

Definition

- Absorption
  $\left(\frac{dN_2}{dt}\right)_{abs} = BN_1 m$
- Stimulated emission
  $\left(\frac{dN_2}{dt}\right)_{e.st} = -BN_2 m$
- Spontaneous emission
  $\left(\frac{dN_2}{dt}\right)_{e.sp} = AN_2$

Semiconductor laser description



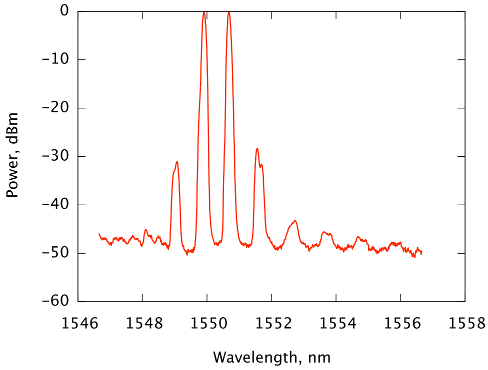
Laser cavity scheme

Definition

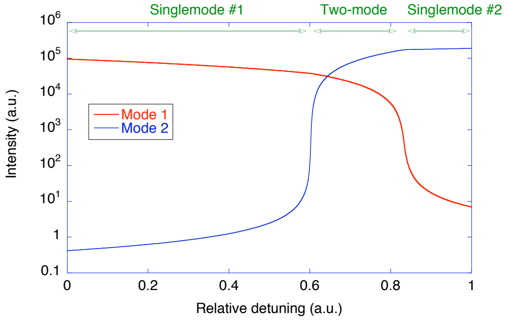
- Laser = Gain medium + laser cavity
- population inversion by pump
- Cavity leakage : laser main output

Summary

Dual mode laser



Multimode laser



Modal competition

[2] L. Chusseau et al., "Four-sections semiconductor two-mode laser for THz generation," *Proc. SPIE* **6343**, 2006, pp. 1097–1106.

Summary

Laser models overview

Quantum simulations

- QED
- Few particles only
- Exact simulations
- Single-mode

Monte-Carlo simulations

- Trade-off between two approaches
- Small to big systems
- Multimode possibility
- Noise intrinsic in system

Rates equations resolution

- Analytic resolution
- No particles number limits
- Multimode possibility
- Fluctuations: Langevin noise

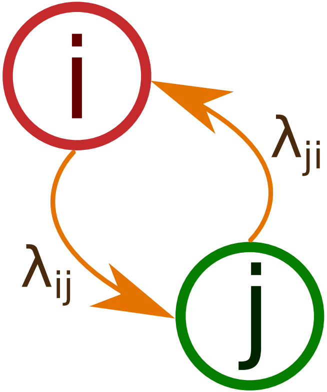
[3] M. Elk, "Numerical studies of the mesomaser," *Phys. Rev. A*, **54**, no. 5, pp. 4351–4358, 1996.

[4] G. P. Puccioni et al., "Stochastic Simulator for modeling the transition to lasing," *Opt. Express*, **23**, no. 3, p. 2369, 2015.

[5] A. Lebreton et al., "Stochastically sustained population oscillations in high-β nanolasers," *New J. Phys.*, **15**, 2013.

[6] L. a. Coldren et al., "Diode Lasers and Photonic Integrated Circuits", *Optical En.*, 1995.

Markov chains



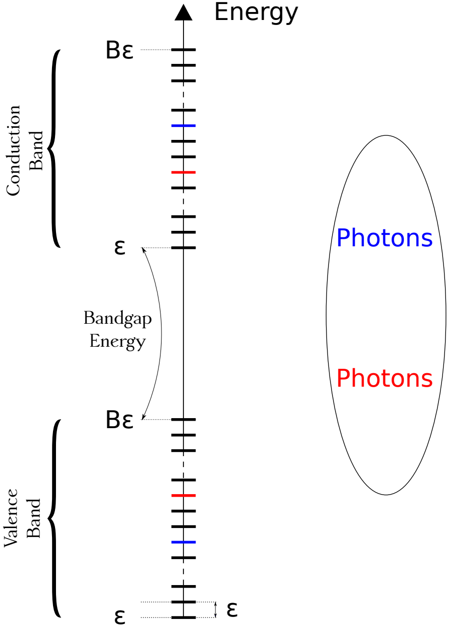
Markov chain simple example

Algorithm 1: Gillespie algorithm : Monte-Carlo simulation by Markov chain

begin
- $t = 0$
- **while** $t < T$ **do**
  - random number $r_t = U(0, 1)$ for the waiting time
  - $\Lambda = \sum_{i \geqslant 1} \lambda_i$
  - $\tau = -\frac{\ln r_t}{\Lambda}$
  - random number $r_e = U(0, 1)$ for the next event
  - $index = \min i : \sum_{j=1}^{i} \lambda_j \geqslant r_e \Lambda$
  - update rates and state
  - $t = t + \tau$

Summary

Model states

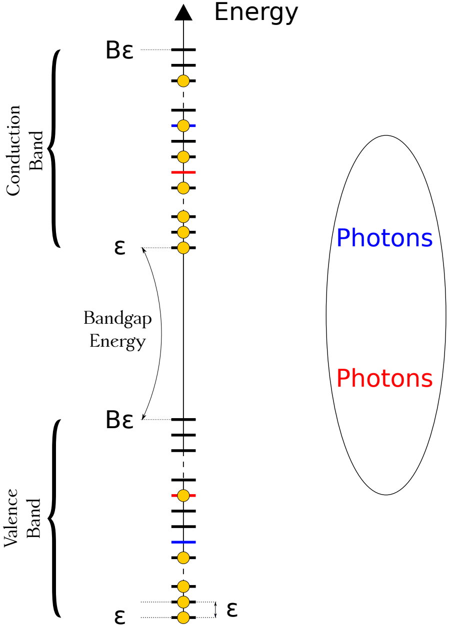


States

- Photon reservoir
- B evenly-spaced energy states in each band
- B electrons in total
- 0 or 1 electron per level
- Photons interact with electrons only at lasing level

Summary

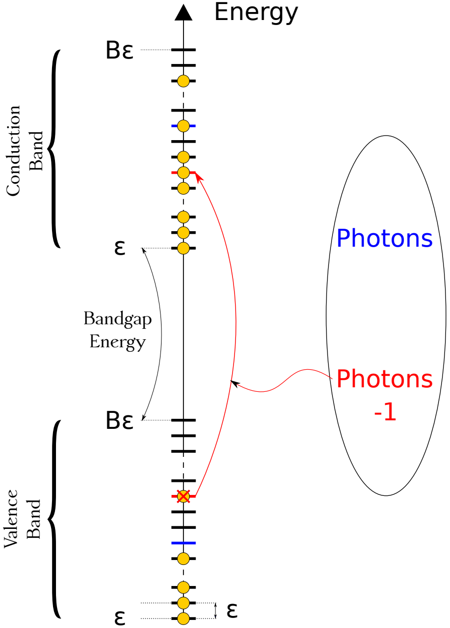
Model events



Events

- Absorption
  $\overline{n_c} n_v m$
- Emission
  $n_c \overline{n_v}(m+1)$
- Cavity output
  αm
- Thermalization
- Steady State :
  Cavity output = pump

Model events



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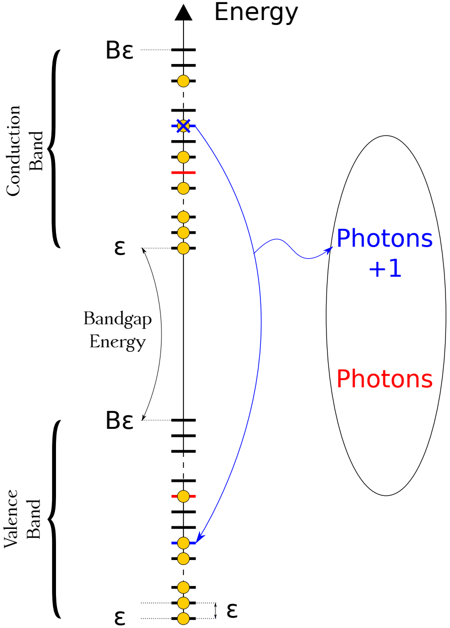

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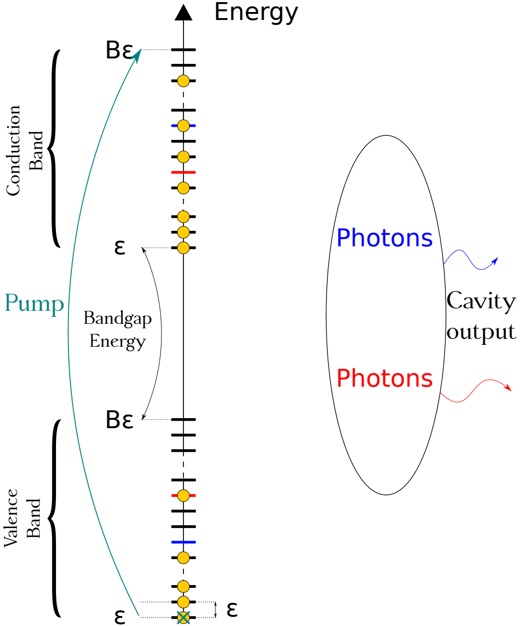

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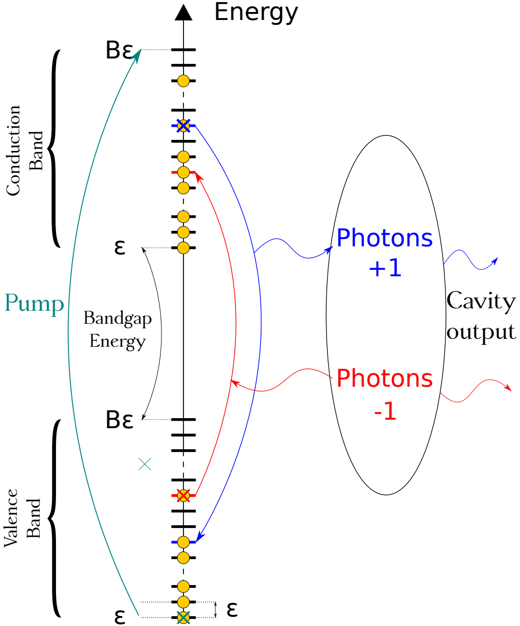

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Summary

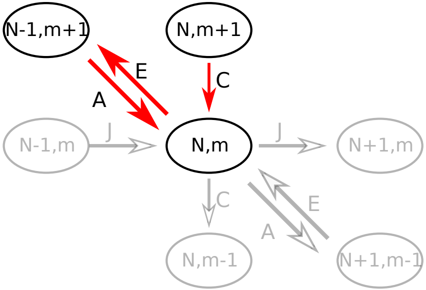
Fermi-Dirac Distribution

$$n_k = \frac{1}{e^{\frac{\epsilon}{k_B T}(k-\mu)} + 1}$$

Fermi-Dirac distribution

Problems

- Occupancy n_k and photon m are independent
- No fluctuation
- Thermal light variance $\Rightarrow$ No laser
- Mean values calculable



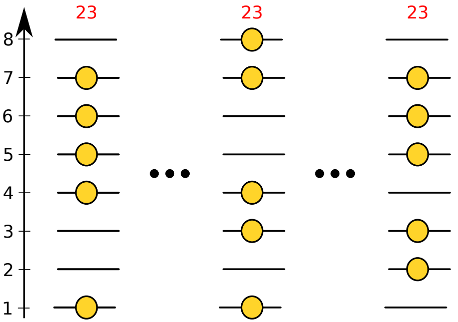
$$\sum_{N=0}^{B} \Pi(N,m)\, n_k^2(N) = \sum_{N=0}^{B} \Pi(N,m+1)\left[\alpha + n_k^2(B-N)\right]$$

Detailed balance at m photons

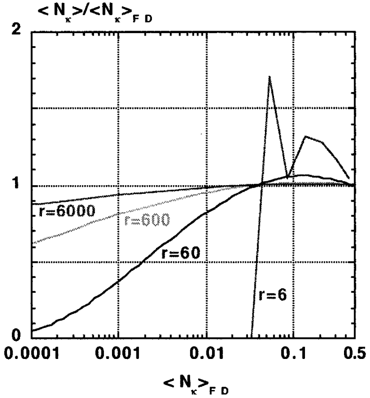
$$\Pi(m) = \frac{\alpha + C_2}{C_1}\Pi(m+1)$$

Summary

Micro-canonical bands repartition



Possible repartitions of electrons for a given energy



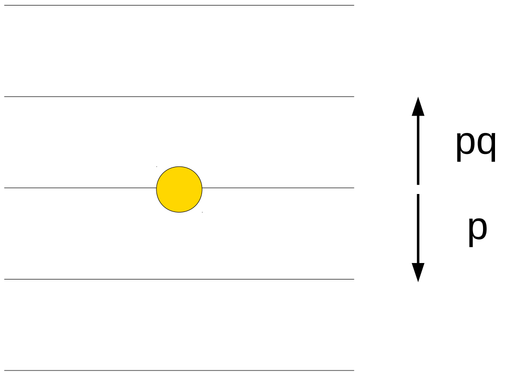
Ratio between Microcanical and FD occupancies for the same energy r

$$m_\kappa(r) = -\sum_{i=\mathbf{1},\mathbf{2},\ldots} (-1)^i \, W[r - i(\kappa + i/2)]$$

[7] J. Arnaud et al., "Illustration of the Fermi–Dirac statistics," *Am. J. Phys.*, **67**, no. 3, pp. 215–221, 1999.

Summary

Microscopic description

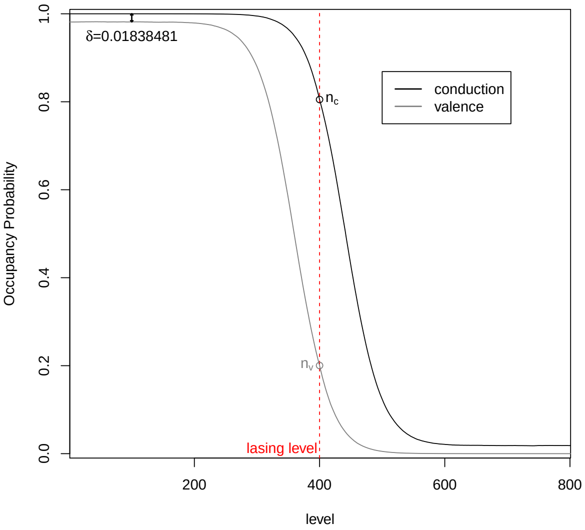


Thermalization event

Definition

- All electron movement simulated
- p $\equiv$ electron movement speed
- Boltzmann thermalization
- q $\equiv e^{\frac{-\epsilon}{k_B T}}$

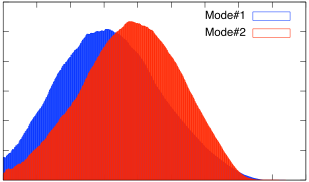
Microscopic application



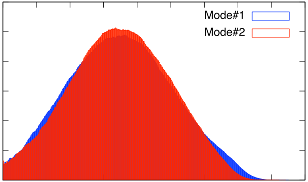
Problems

- Occupancy shift
- Pump values biased
- $> 10^5$ thermalization events per photon event
- Very time consuming

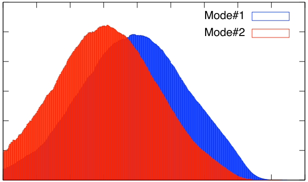
Microscopic modal competition



$\alpha_L = 0.615$



$\alpha_L = 0.616$



$\alpha_L = 0.617$

Stable dual-mode solution exist but only a very tiny range of laser parameters

[8] L. Chusseau et al., "Monte Carlo modeling of the dual-mode regime in quantum-well and quantum-dot semiconductor lasers," *Opt. Express*, **22**, no. 5, pp. 5312–5324, 2014.

Summary

Canonical ensemble

Hypothesis

We may use canonical occupancy at laser level.

$$n_k(N,q) = \frac{Z(N-1,q)q^k}{Z(N,q)}\left[1 - n_k(N-1,q)\right]$$

Canonical Occupancy

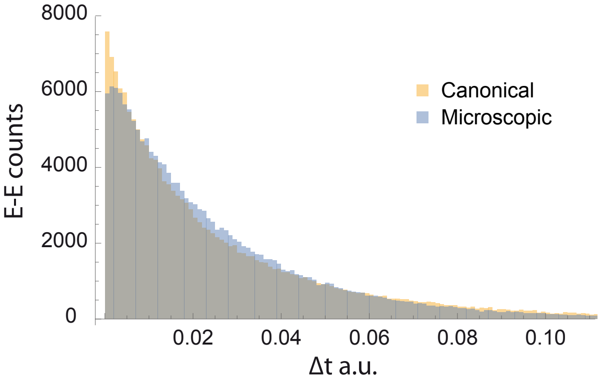
$$Z(N,q) = \prod_{i=1}^{N} \frac{q^{i-1} - q^B}{1 - q^i}$$

Partition function

$$n_k(N+1,q) = \frac{(1-q^{N+1})q^k}{q^N - q^B}\left[1 - n_k(N,q)\right]$$

Recursive occupancy formula

Canonical vs. Microscopic difference

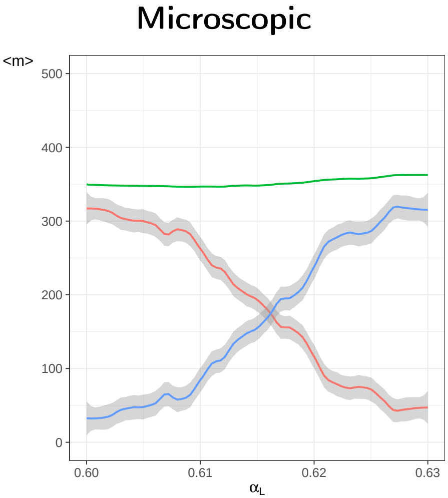


Distributions of double emission events

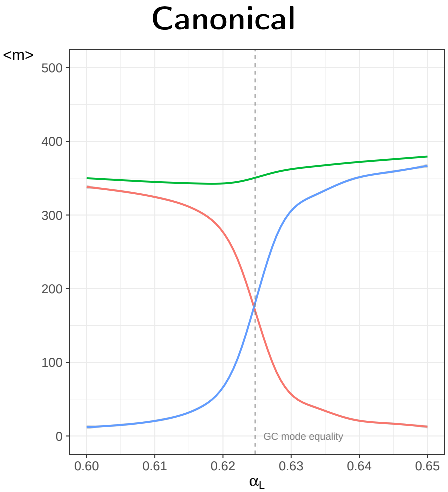
Double emission events

- Canonical : Exponential distribution
- Microscopic : Non exponential distribution
- $\exists$ Solution to restrain fast E-E within the Markov framework

Dual-mode photon statistics



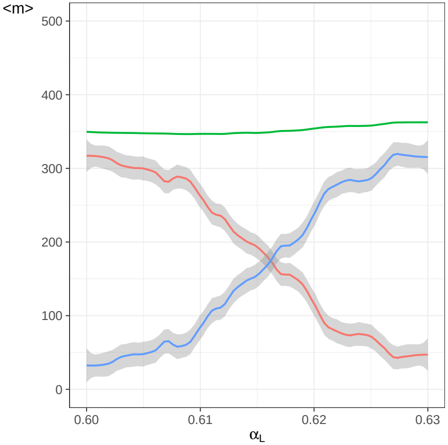
Photon statistic for microscopic at pump rate 210



Photon statistic for canonical at pump rate 210

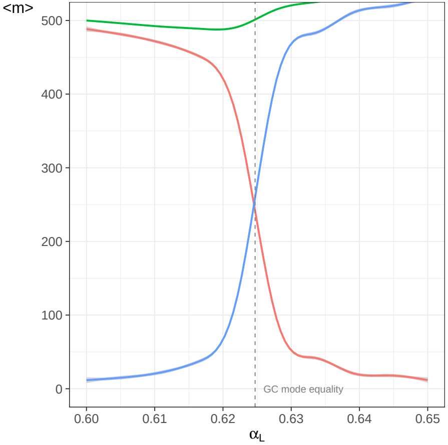
Dual-mode photon statistics

Microscopic



Photon statistic for microscopic at pump rate 210

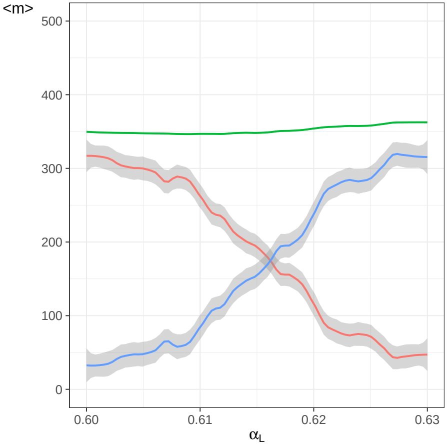
Canonical



Photon statistic for canonical at pump rate 300

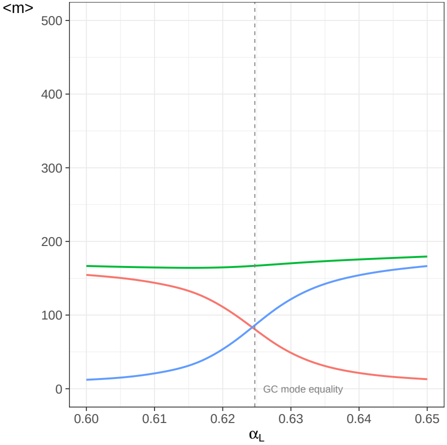
Dual-mode photon statistics

Microscopic



Photon statistic for microscopic at pump rate 210

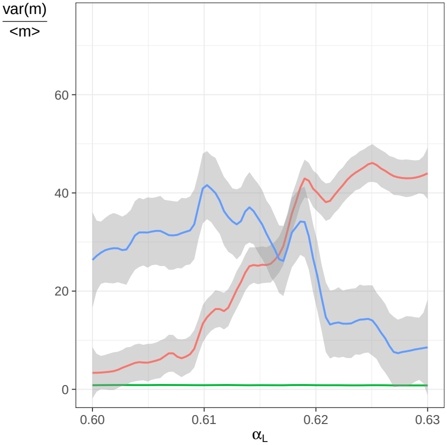
Canonical



Photon statistic for canonical at pump rate 100

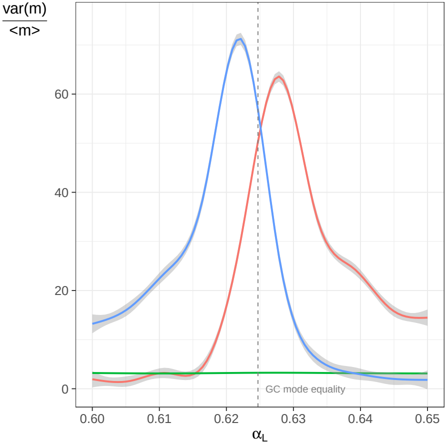
Bimode photon Fano factor

Microscopic



Photon Fano factor for microscopic at pump rate 210

Canonical



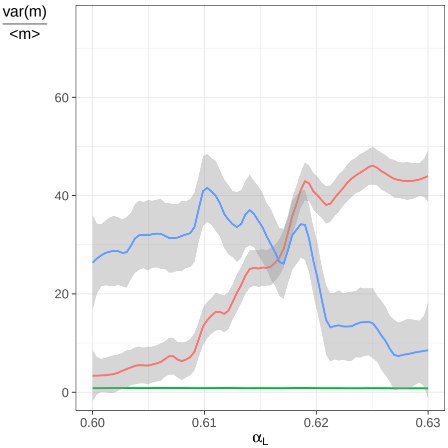
Photon Fano factor for canonical at pump rate 210

Laser Markov

Arthur Vallet

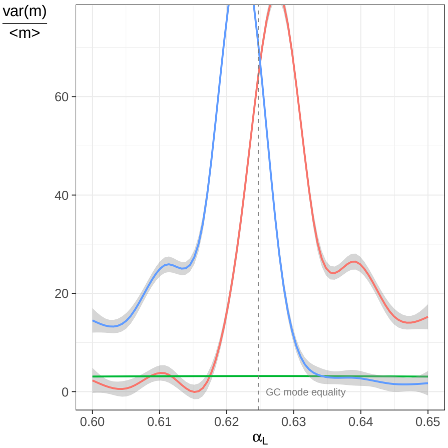
Bimode photon Fano factor

Microscopic



Photon Fano factor for microscopic at pump rate 210

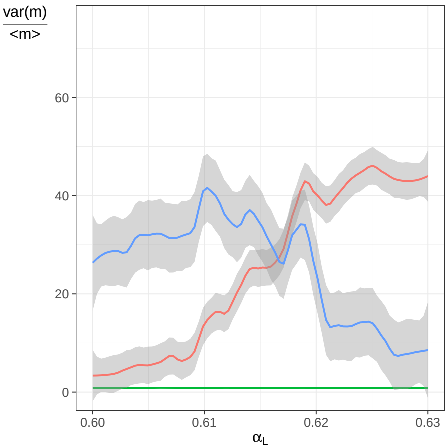
Canonical



Photon Fano factor for canonical at pump rate 300

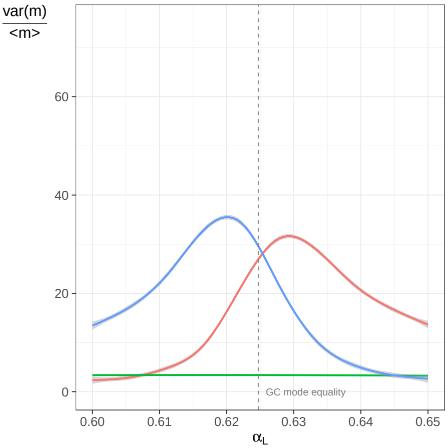
Bimode photon Fano factor

Microscopic



Photon Fano factor for microscopic at pump rate 210

Canonical



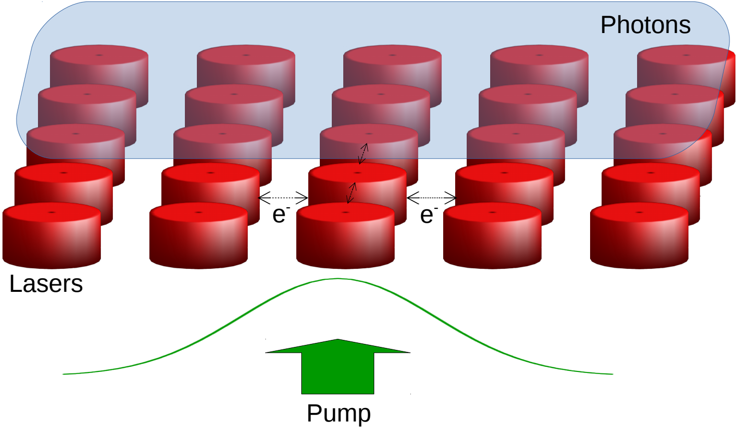
Photon Fano factor for canonical at pump rate 100

Conclusion

Monte-Carlo semiconductor laser simulation based on Markov chain

- requires electron presence at lasing levels → different statistical ensembles.
- **Grand-canonical**: thermal light → nonsense for a laser.
- **Micro-canonical**: accurately, but considers only closed systems.
- **Microscopic**: take thermalization into account.
  Satisfactory from physical point of view but was very slow.
- **Canonical**: shortcut in Markov chain
  Speed up the program from $1h$ to $1s$ with the same physical accuracy.

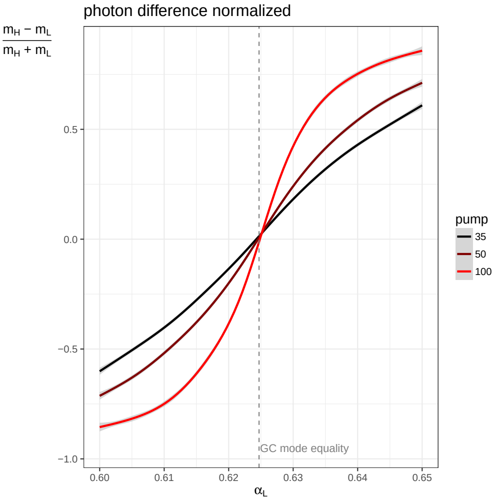
Future



2D Semiconductor laser model

appendix

Modal competition

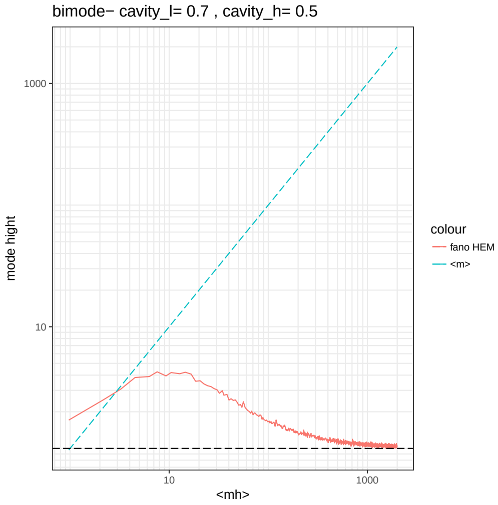


Canonical modal competition for different cavity output and pump rates

Modifications

- CPU time:
  1 hour $\rightarrow$ 1 sec
- No occupancy congestion
- Microscopic convergence $p \rightarrow \infty$

Single-mode threshold

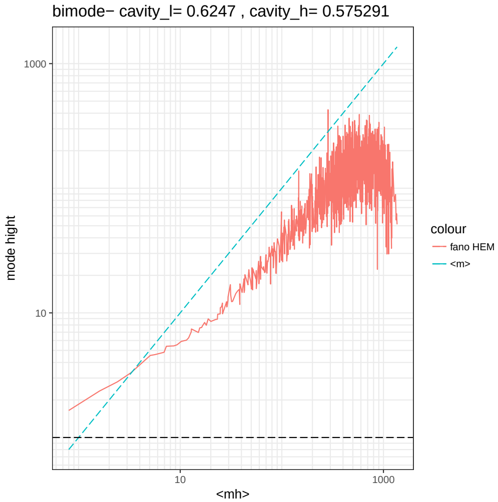


Fano factor of a quasi-single-mode laser

Threshold definition

- Mode extinction
  extinction frequency
  extinction length
- Fermi level maximum
- Fano maximum

Threshold bi-mode



Fano factor of dual-mode with equality of photons mean

Threshold definition

- Mode threshold
- laser threshold
- noise dependence of modal competition